KGÜ 8.5

A32

a) i) $\bar{x} = \frac{1}{12}(11+9+14+7+8+10+12+7+15+13+9+11) = \frac{1}{12} \cdot 126 = 10.5$

ii)

$x_{(1)}$	$x_{(2)}$	$x_{(3)}$	$x_{(4)}$	$x_{(5)}$	$x_{(6)}$	$x_{(7)}$	$x_{(8)}$	$x_{(9)}$	$x_{(10)}$	$x_{(11)}$	$x_{(12)}$
7	7	8	9	9	10	11	11	12	13	14	15

$n=12 \Rightarrow \tilde{x}_{med} = \frac{1}{2}\left(x_{(n/2)} + x_{(n/2+1)}\right) = \frac{1}{2}(x_{(6)} + x_{(7)}) = 10.5$

iii) unteres Quartil → gilt ein Wert x, wenn mind. 25% $\leq x$ und mind. 75% der Datenpunkte $\geq x$ sind

$n \cdot q = 12 \cdot 0.25 = 3 \in \mathbb{N}$

$$\tilde{x}_{0.25} = \frac{1}{2}\left(x_{(0.25n)} + x_{(0.25n+1)}\right) = \frac{1}{2}(x_{(3)} + x_{(4)}) = 8.5$$

iv) $s_x^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x}_i)^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - (\bar{x})^2 =$

$= \frac{1}{12}(11^2+9^2+14^2+7^2+8^2+10^2+12^2+7^2+15^2+13^2+9^2+11^2) - 10.5^2 =$

$= \frac{1}{12}(121+81+196+49+64+100+144+49+225+169+81+121) - 110.25 =$

$= \frac{1400}{12} - 110.25 = \frac{1400}{12} - \frac{441}{4} = \frac{1400-133}{12} = \frac{77}{12}$

v) $s_x = \sqrt{s_x^2} = \sqrt{\frac{77}{12}}$

b) $\bar{y} = 2\bar{x} - 1 = 2 \cdot 10.5 - 1 = 20$

für $x = (x_1, \dots, x_{12})$

$y = (y_1, \dots, y_{12})$

$s_y^2 = \mathrm{var}(y) = \mathrm{var}(2x-1) = \mathrm{var}(2x) = 2^2\,\mathrm{var}(x) = 4 \cdot s_x^2 =$

$= 4 \cdot \frac{77}{12} = \frac{77}{3}$

A33

a)

$x_{(1)}$	$x_{(2)}$	$x_{(3)}$	$x_{(4)}$	$x_{(5)}$	$x_{(6)}$	$x_{(7)}$	$x_{(8)}$	$x_{(9)}$	$x_{(10)}$
0.3	0.5	0.5	0.8	0.8	1.0	1.2	1.5	1.5	1.5

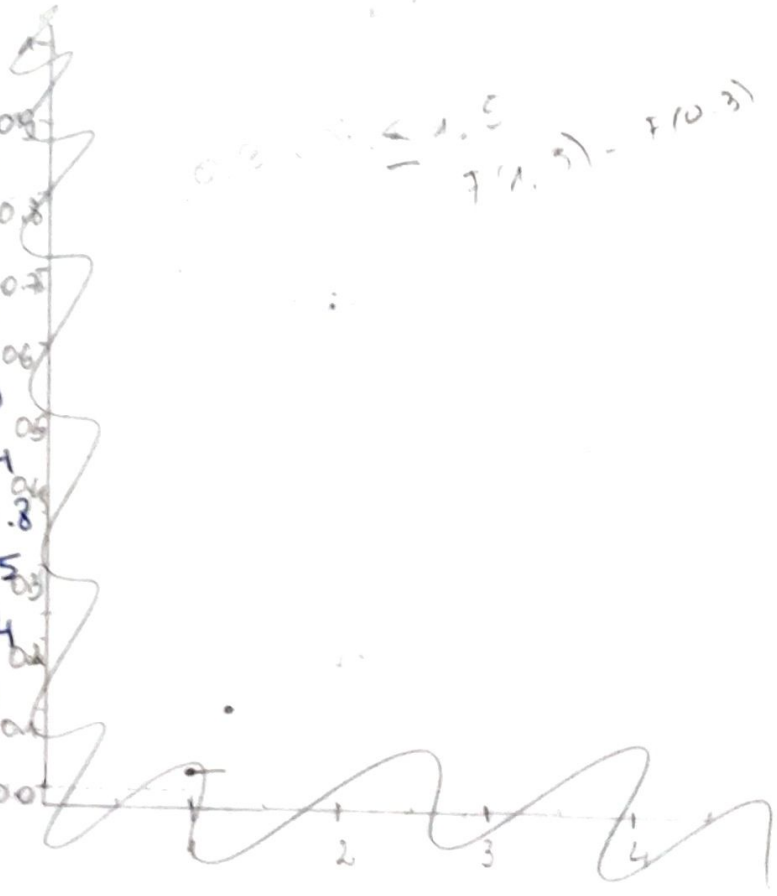
$x_{(11)}$	$x_{(12)}$	$x_{(13)}$	$x_{(14)}$	$x_{(15)}$	$x_{(16)}$	$x_{(17)}$	$x_{(18)}$	$x_{(19)}$	$x_{(20)}$
1.5	1.9	2.4	2.4	2.8	3.5	3.5	3.5	4.0	4.1

$n=20$

Werte	0.3	0.5	0.8	1.0	1.2	1.5	1.9	2.4	2.8	3.5	4.0	4.1
abs. H.	1	2	2	1	1	4	1	2	1	3	1	1
rel H.	0.05	0.1	0.1	0.05	0.05	0.2	0.05	0.1	0.05	0.15	0.05	0.05
kum. rel H.	0.05	0.15	0.25	0.3	0.35	0.55	0.6	0.7	0.75	0.9	0.95	1

b)

$$\hat{F}_{20}(x)=\begin{cases} 0, & x<0.3 \\ 0.05, & 0.3\le x<0.5 \\ 0.15, & 0.5\le x<0.8 \\ 0.25, & 0.8\le x<1 \\ 0.3, & 1\le x<1.2 \\ 0.35, & 1.2\le x<1.5 \\ 0.55, & 1.5\le x<1.9 \\ 0.6, & 1.9\le x<2.4 \\ 0.7, & 2.4\le x<2.8 \\ 0.75, & 2.8\le x<3.5 \\ 0.9, & 3.5\le x<4 \\ 0.95, & 4\le x<4.1 \\ 1, & 4.1\le x \end{cases}$$



c) ≤ 1

$\hat{F}_{20}(1) = 0.3$

oder

$\hat{F}_{20}(1) = \frac{1}{20} \sum_{i=1}^{20} 1\{x_i \leq 1\} =$ x_i = ursprünglichen ungeordneten Daten

$= \frac{1}{20}(1+1+0+1+0+1+0+0+0+0+0+0+0+0+1+1+0+0+0+0+0)$

$= \frac{6}{20} = 0.3$

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zwischen 1.2 und 2.4:

$\hat{F}(2.4) - \hat{F}(1.2) = 0.7 - 0.35 = 0.35$

d) $K_1 = [0.3, 0.8]$; $K_3 = (1.3, 1.9]$; $K_5 = (3.0, 4.1]$

$K_2 = (0.8, 1.3]$; $K_4 = (1.9, 3.0]$;

i	1	2	3	4	5
Klasse K_i	$[0.3, 0.8]$	$(0.8, 1.3]$	$(1.3, 1.9]$	$(1.9, 3.0]$	$(3.0, 4.1]$
abs H.	5	2	5	3	5
rel H.	0.25	0.1	0.25	0.15	0.25
b_i	0.5	0.5	0.6	1.1	1.1
h_i	0.5	0.2	0.42	0.17	0.23

134 1.0, 2.5, 2.0, 3.5, 1.0

a)

Ausprägung	1.0	2.0	2.5	3.5	Σ
abs. H.	2	1	1	1	5
rel. H.	2/5	1/5	1/5	1/5	1

b)

$x_{(1)}$	$x_{(2)}$	$x_{(3)}$	$x_{(4)}$	$x_{(5)}$
1.0	1.0	2.0	2.5	3.5

$x_{min} = x_{(1)} = 1$

$x_{max} = x_{(5)} = 3.5$

$n = 5$ ungerade $\Rightarrow x_{med} = x_{(\frac{5+1}{2})} = x_{(3)} = 2.$

unteres Q: $n \cdot p = 5 \cdot 0.25 = 1.25 \notin \mathbb{N} \Rightarrow \tilde{x}_{0.25} = x_{(\lfloor n \cdot p \rfloor + 1)} = x_{(2)} = 1$

oberes Q: $n \cdot p = 5 \cdot 0.75 = 3.75 \notin \mathbb{N} \Rightarrow \tilde{x}_{0.75} = x_{(\lfloor n \cdot p \rfloor + 1)} = x_{(4)} = 2.5$

IQR: $\tilde{x}_{0.75} - \tilde{x}_{0.25} = 2.5 - 1 = 1.5$

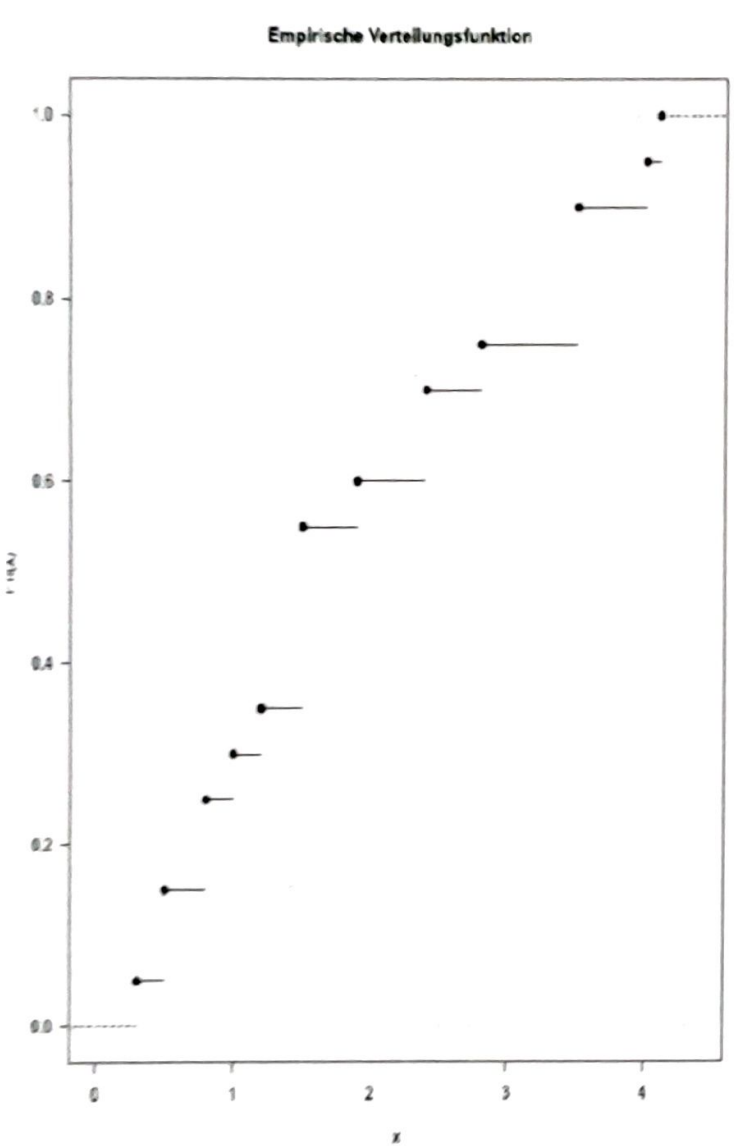
0.2-Quartil: $n \cdot p = 5 \cdot 0.2 = 1 \in \mathbb{N}$

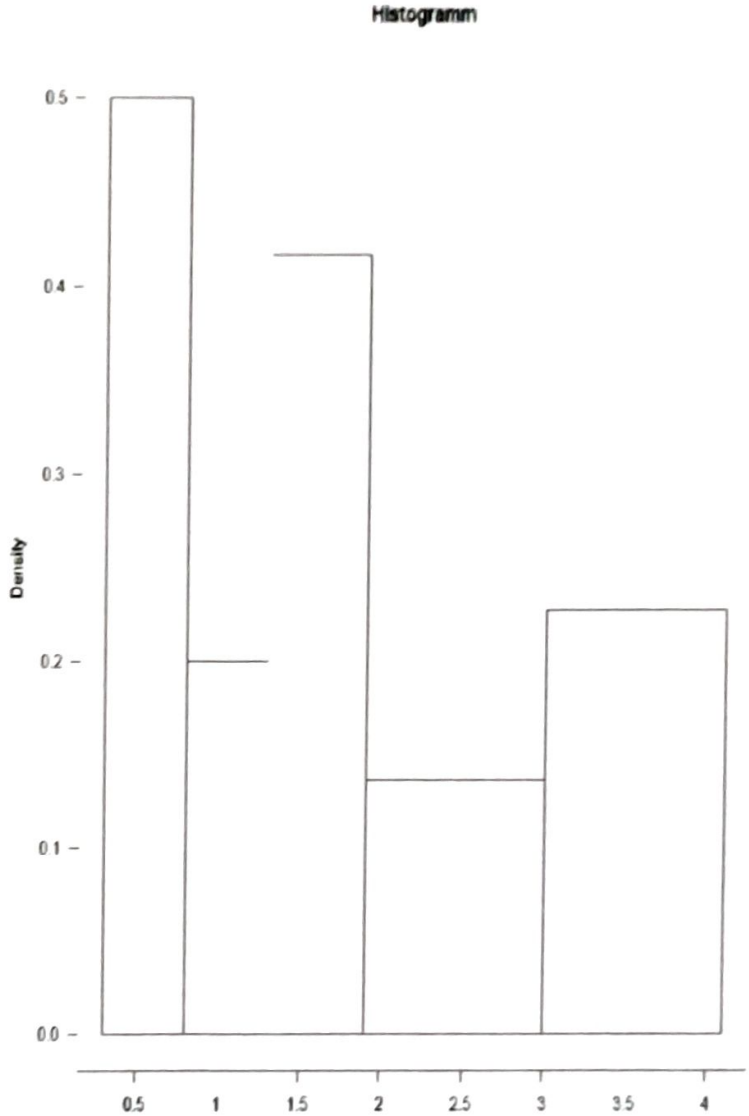
$\Rightarrow \tilde{x}_{0.2} = \frac{1}{2}(x_{(np)} + x_{(np+1)}) = \frac{1}{2}(x_{(1)} + x_{(2)}) = \frac{1}{2}(1+1) = 1$

c)

Ausprägung	1.0	2.0	2.5	3.5
rel H.	2/5	1/5	1/5	1/5
kum. H.	2/5	3/5	4/5	1

$$\hat{F}_5(x) = \begin{cases} 0, & x < 1 \\ 2/5, & 1 \leq x < 2 \\ 3/5, & 2 \leq x < 2.5 \\ 4/5, & 2.5 \leq x < 3.5 \\ 1, & 3.5 \leq x \end{cases}$$


Empirische Verteilungsfunktion
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
x


Histogramm
Density
0.5
0.4
0.3
0.2
0.1
0.0
0.5
1
1.5
2
2.5
3
3.5
4